A morphological dominant points detection and its cellular implementation

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Goal

This work proposes a new dominant (interest) points detector based on mathematical morphology tools.

- What is an interest point ?
- Why and How using mathematical morphology tools ?

What is an interest point ?

There are two kinds of interest points which are :

- corner points (L-points with more or less important angle)
- junction points (T-points)

Such points are useful for :

- image matching : correlation between several images
- registration : research of image
- image synthesis : model 3D objects in 2D

Why and How using the mathematical morphology tools ?

A set of mathematical tools widely used on pattern recognition originally defined by Serra & Matheron.

opening of X by B, $X \circ B = (X \ominus B) \oplus B$
where X denotes a compact set and B the structuring element.

- Well adapted to SIMD parallel machine
- Experience of algorithms implemented with mathematical morphology on the lab and many of them were mathematically proved (Manzanera).

Dominant Points

The three main categories of interest points detectors found on literature are :

- Chain code (HORAUD, DERICHE, ASADA & BRADY)
- Signal proceeding (MORAVEC, HARRIS, BEAUDET)
- Theoretical models

Some works were made with mathematical morphology (ZHANG & ZHAO,1995).

Zhang & Zhao corner points detector

Principle :

- Filling closed curves X
- detect corner points with :

$$[(X \setminus (X \circ D(n))) \cup (X^c \setminus (X^c \circ D(n)))] \cap (X - (X \ominus D(n)))$$

((Residue of X) OR (Residue of X^c)) AND (Thinning of X)

advantage & disadvantage of this process

advantage	disadvantage
• Use particularity of SIMD parallel machine • No floating calculation as with chain code techniques • Noise is reduce with smoothing effect of the algorithm	• Only detect convex corner points • Don't work with opened curves (because of filling) • Sensibility with structuring element size and connexity

Some Definitions

Let I be an image in grayscale t

$$\begin{array}{rccc} I : & \mathbb{Z}^2 & \longrightarrow & [[0,255]] \\ & p & \longmapsto & I(p) \end{array}$$

$$I_t = \{p \in \mathbb{Z}^2, I(p) \geq t\}$$

$$I(p) = \max_{t \in [[0,255]]} \{I_t(p)\}$$

Let $S \in I$ a shape of I, a subset of I,

$\overset{\circ}{S}$ be the contour of the shape S,

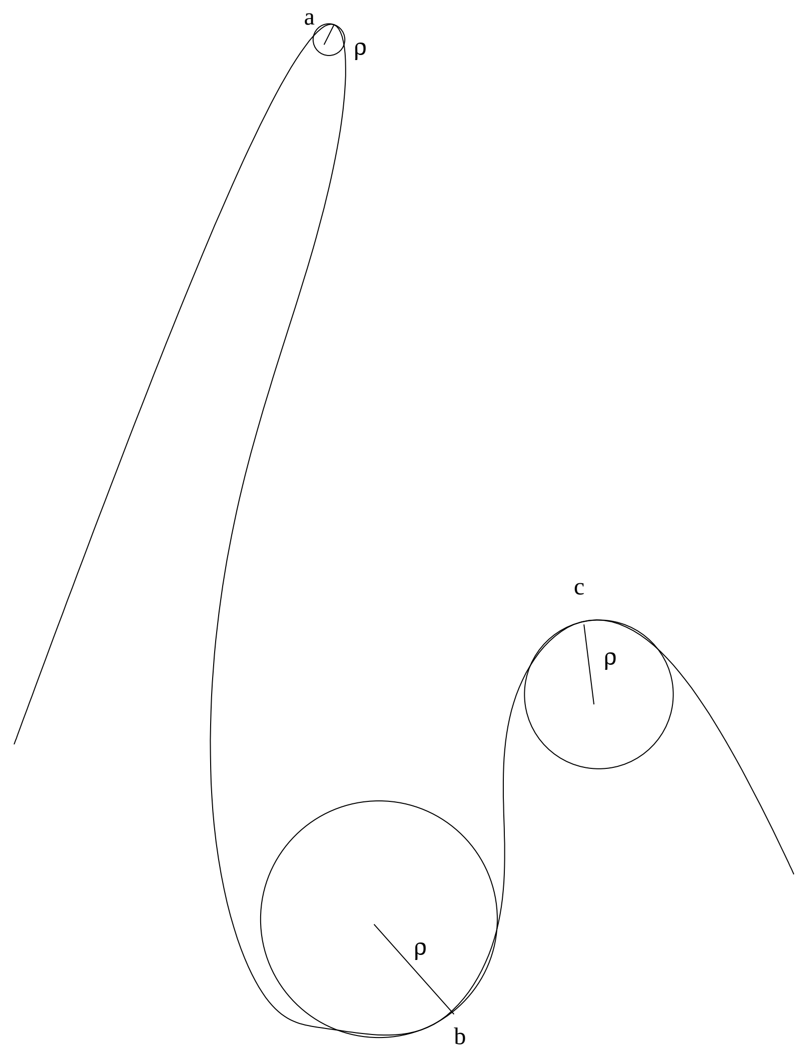
$D(n)$ be a disk structuring element (symmetric),

R_p be the curvature radius at point $p \in \overset{\circ}{S}$,

$\rho_{I(p)}$ be the radius of the maximal inscribed circle in S.

It comes :

measure of local curvature



<u>Asada & Brady</u>

$$R_p = \frac{\frac{\partial^2 y}{\partial x^2}}{[1 + (\frac{\partial y}{\partial x})^2]^{3/2}} \Leftrightarrow \rho_I(p) = \frac{1}{R_p + 1} \tag{1}$$

<u>Yan & Chen</u>

If we have $S \circ D(n)$ then $inf\{R_p | R_p > 0\} \geq n$, where inf denotes inferior (2)

<u>Zhang & Zhao</u>

- If $n > R_p$ then $p \notin S \circ D(n)$ (3) [(2)⇒(3)]
- If $n > \rho_{I(p)}$ then $p \notin S \circ D(n) \Rightarrow p \in S - (S \circ D(n))$ (4)

⇒ [with (3) and (4)] We can detect dominant points with residuals operations $(S - (S \circ D(n)))$

Skeleton

Skeleton is defined according to the analogy of grass fires of BLUM.

Properties :

- Reconstructability : homotopy, preserve topology
- Rotation-invariance : $\frac{\pi}{2}$ angle multiple
- Noise immunity
- Thinness : one-pixel-thickness and mediality

According to SERRA, in the discrete plane, these requirements become mutually incompatible so we have to make compromise between them.

Residuals + directional gaps = skeleton

In practical, algorithm that computes skeleton of binary patterns is obtained by applying directional erosions, while retaining those pixels that introduce disconnections. (Cardonner & Thomas)

$$S(X) = X - \backslash\{p \in X, p \in S \circledast \alpha \; and \; p \notin S \circledast \beta\}$$

residuals directional graps

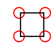
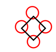
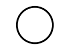
with :

$$B = (H, M), \quad H \cap M = \emptyset, \quad \check{B} = -B$$

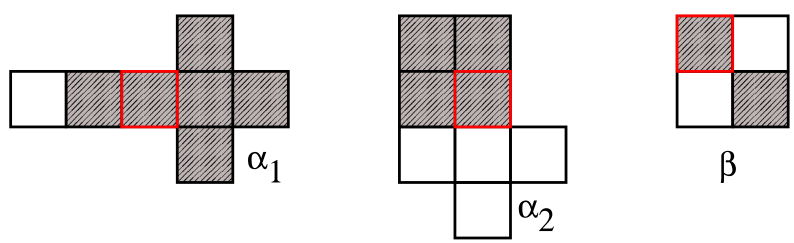
$$X \circledast (H, M) = (X \ominus \check{H}) \cap (X^c \ominus \check{M})$$

where $X \circledast B$ is the Hit-or-Miss transform of X by B.

Which skeleton use for our works ?

we want			
MB1DIR Jan & Chin	.	+	+
MB1FP Latecki	×	+	×
MB2 Guo & Hall	.	.	.
MB1Hyb Manzanera	×	+	.

which MB1 is a skeleton with (α_1, β) , MB2 with (α_2, β) and DIR or FP is the functional mode (directional or full parallel). Hyb (for hybride) is a mixed directional and full parallel skeleton process.



binary to grayscale process

Let ω be a growing binary operator and Ω a grayscale operator, X and Y compact set and $B(n)$ a structuring element of size n. Then we have :

Binary operator	Grayscale operator
X^C complement	$255 - X(t)$ inversion
$X \cap Y$ intersection	$\min_t \{X(t), Y(t)\}$
$X \cup Y$ union	$\max_t \{X(t), Y(t)\}$
$X \ominus B(n)$ erosion	$\min_{y \in B} \{X(t+y) - B(y)\}$
$X \oplus B(n)$ erosion	$\max_{y \in B} \{X(t-y) + B(y)\}$
$X \circ B(n)$ opening	$(X \ominus B(n)) \oplus B(n)$
$X \bullet B(n)$ closing	$(X \oplus B(n)) \ominus B(n)$

Residuals operations

$$r(I_t) = I_t \setminus \omega(I_t)$$

$$\begin{aligned} R(I) &= \sum_t r(I_t) \\ &= I \setminus \Omega(I) \end{aligned}$$

with :

$$\Omega(I) = \sum_t \omega(I_t)$$

which we can rewrite with min, max operators under conditions.

But take attention to :

$$X \setminus Y = \begin{cases} 0\, if\, X(t) < Y(t), \\ X(t) - Y(t)\, otherwise \end{cases}$$

Interest function

Let $\varphi_{I_t}(p)$ be the interest function associate with contour point p in binary image I_t and define as :

$$\varphi_{I_t}(p) \propto \rho_{I_t}(p)\,,\; p \in \mathring{I}_t(S)$$

Then let $\phi_I(p)$ be the interest function associate with contour point p in grayscale image I and define as :

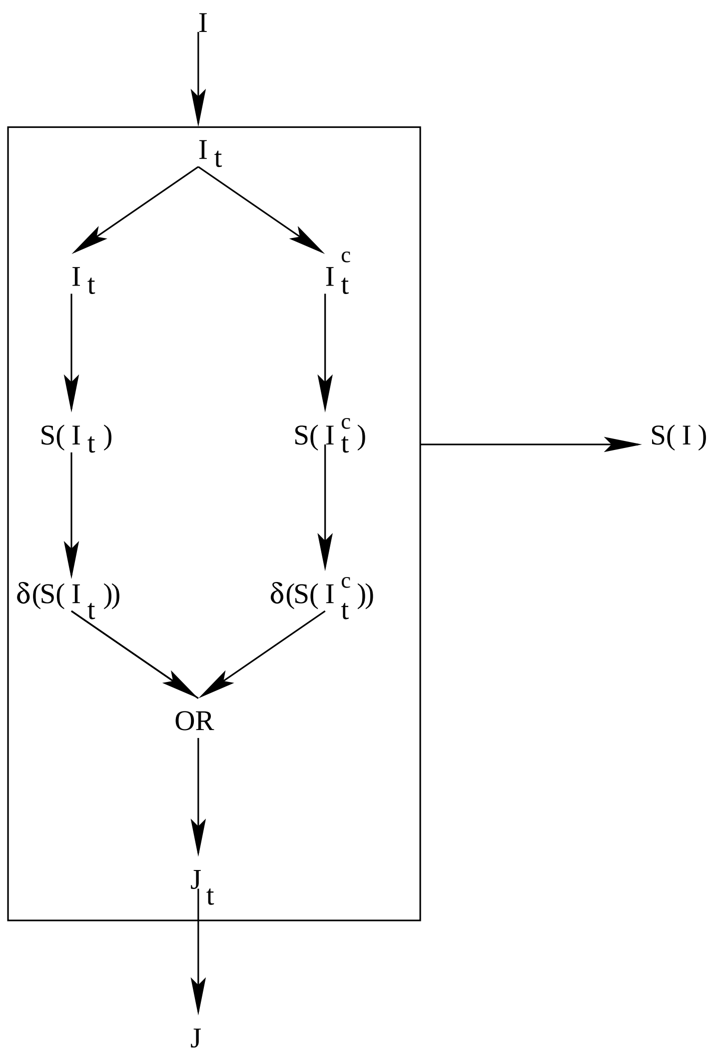
$$\begin{array}{rccl} \phi: & E & \rightarrow & K \in [0, 255] \\ & p & \mapsto & \displaystyle\sum_{t \in K} \varphi_{I_t}(p) \end{array}$$

Result image is a grayscale image where each point has a value corresponding to its "interest" on the original image.
More or less dominant point could be detected with thresholding this result image.

2 questions :

- How to exactly determine the thresholding level ?
  it depends on the use of dominant points and on the original image (artificial or natural one)
- Can we compute interest function directly on grayscale image ?
  It seems that compute a skeleton in grayscale is equivalent to make a sum of each skeleton computes for each gray level.

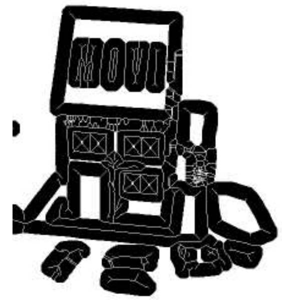
Implementation



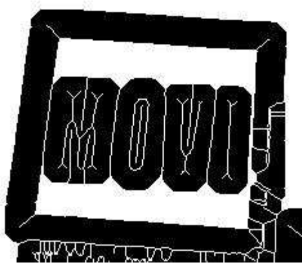
results and performances

original image

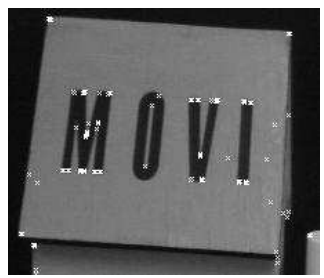
binary image



skeleton/exoskeleton



skeleton detail



result detail

ZhangZhao result

conclusion

- SIMD parallel machine
  adapted, fast, robust
- other architecture
  more complex, repetitive computing (skeleton) but comparable results
- We can detect
  terminal points
  junction points with exo-skeleton (double, triple or quadruple)
  corner points

References

[Asa 86] H. Asada & M. Brady, *The curvature primal sketch*, IEEE transactions on Pattern Analysis and Machine Intelligence, 8(1), 2-14, 1986.

[Car 97] R. Cardoner & F. Thomas, *Residuals + directional gaps = skeletons*, Pattern recognition letters, 18, 343-353, 1997.

[Chen 89] M-H. Chen & P-F. Yan, *A multiscale appoach based on morphological filtering*, IEEE transactions on Pattern Analysis and Machine Intelligence, 11, 694-700, 1989.

[Man 00] A. Manzanera, *Vision artificielle rtinienne*, Thse de doctorat, Ecole Nationale Suprieure des Tlcommunications, 2000, Paris, France, Novembre.

[Man 02] A. Manzanera & Thierry M. Bernard, *Derivation of a coherent collection of 2D parallel thinning algorithms from* *desired topological and metrical properties*, Elsevier Science, Octobre 2002.

[Ser 82] J. Serra, *Image analysis and mathematical morphology*, New York, Academic Press, 1982.

[Zha 95] X. Zhang & D. Zhao, *A morphological algorithm for detecting dominant points on digital curves*, SPIE Proceedings, Non Linear Image Processing VI, Vol. 2424, 372-383, 1995.